Best results are obtained when simultaneous readings are made at a near-by station of known elevation at the base of the hill or mountain.

The average of three or more simultaneous readings taken at 15 minutes or half hour intervals ordinarily will give better results than can be obtained from single readings.

If the base station is located near by and all readings are accurately made it should be possible to figure the difference in elevation very closely, using reduction tables such as those published in "Smithsonian Meteorological Tables." It is important to record the attached thermometer and shade air temperatures also, for use in correcting and reducing the readings.

In the Tropics where the air is warm and light one inch difference in air pressure is equivalent roughly to 1,000 feet difference in altitude, while in the temperate zone where the air is cooler and denser an inch difference in pressure may equal only 850 or 900 feet difference in altitude. For this reason the arbitrary fixed altitude scales on aneroids are not reliable for varying temperature conditions.

If it is not possible to take simultaneous readings at a near-by base station, the pressure at the lower station must be estimated. Under these conditions less accurate results will be obtained.

The accompanying plate (No. 2) shows typical winter pressure curves for temperate zone and tropical climates. The large irregular pressure fluctuations in the temperate zone make it extremely difficult to determine elevations accurately from barometric readings unless simultaneous readings at a near-by base station can be made.

In the Tropics the barometric pressure is so constant, except for regular, well-marked diurnal fluctuations, that the sea-level pressure can be estimated closely, and fairly good altitude determinations can be made without taking simultaneous pressure readings at a near-by base station.

The following results were obtained in the Canal Zone, elevations being determined from the average of six mercurial barometer readings taken simultaneously at the upper and lower stations.

Station.	Elevation by barometer.	Actual elevation by triangulation.
	Feet.	*Feet.*
Ancon Hill	659	654
Cerro Gordo	965	972

It will be seen that the elevations obtained by barometer were off less than 1 per cent (assuming the elevations by triangulation to be correct).

Individual readings varied but slightly from the mean of all readings, as may be seen from the following table:

Time.	Corrected station pressure.		Indicated difference.	Indicated altitude of Ancon Hill.[1]
	Ancon.	Ancon Hill.		
	Inches.	*Inches.*	*Feet.*	*Feet.*
1:45 p. m.	29.638	29.078	562	654
2:00 p. m.	29.641	29.077	568	660
2:15 p. m.	29.641	29.078	567	659
2:30 p. m.	29.640	29.071	573	665
2:45 p. m.	29.641	29.083	559	651
3:00 p. m.	29.638	29.069	573	665
3:15 p. m.	29.631	29.067	565	657
Average of all readings				659

[1] Difference in feet plus 92 feet (elevation of Ancon Station).

These determinations were made under the most favorable conditions, the base station at Culebra being not more than 2 miles distant from Cerro Gordo, and the base station at Ancon less than a mile distant from Ancon Hill. Actual field work would often have to be performed under less favorable conditions, with base stations farther distant or unavailable, in which case less accurate results would be obtained.

COMPARISON OF SNOW-BOARD AND RAINGAGE-CAN MEASUREMENTS OF SNOWFALL.

By ROBERT E. HORTON.

[Voorheesville, N. Y., Mar. 17, 1920.]

The unusual accumulation of snow in eastern New York afforded an opportunity for comparison of the accuracy of measurements of snowfall by two different methods in common use. The rain gage overflow can and the snow board were both exposed on the ice near the center of a pond at the author's laboratory, the pond being about 100 feet wide and several hundred feet in length, in an easterly and westerly direction. The north slope to the pond ranges from 10 to 20 feet per hundred, and the pond is bordered on the north by occasional trees and brush. The south bank is abrupt and wooded. Snow drifts on the pond surface only on rare occasions.

The snow board used was that devised by the author, consisting of a sheet of white beaver board, about 16 inches square, with a layer of cotton flannel tacked on to the surface of the beaver board, nape uppermost. After each reading was taken, the snow board was cleaned and dried, and laid on the surface of the newly fallen undisturbed snow. In all the storms recorded in the table, the snow fell mainly during wind, and at a large angle to the vertical, often approaching the horizontal.

Comparing the results as shown in column 5, it will be noted that the average depth of snowfall, as determined by the water equivalent, is 16 per cent more than that determined from measurements taken in the overflow can of the rain gage. In taking the readings, the gage can was first weighed, the snow then removed therefrom, and a sample cut out of the snow on the snow board by inverting the gage can over the snow board, like a cookie cutter, then picking up gage can and snow board together, so as to get a perfect sample in the gage can. The gage can was then again weighed. An accurate torsion balance was used, making possible in all cases to determine the water equivalent of the snow to the nearest thousandth of an inch of water.

It will be noted that in very light snow flurries, the amount caught on the snow board might be equal to or less than that caught in the gage can. In all heavier snows, the catch on the snow board was greater, and by a fairly consistent percentage. Much of this snow fell when the temperature was about 32°, and while the type of snow board used was specially designed to simulate a snow surface, and prevent melting, the results indicate that in very light snow flurries the snow board may give deficient results. The most significant result is, however, the fact that for two months taken as a whole, the excess indicated by the snow board is 16 per cent as compared with gage-can measurements. This on a total winter's snow precipitation of 12 inches amounts to roundly 2 inches, a fact which, if generally true, helps to afford an explanation of the apparent deficiency of winter water losses, often observed by comparison of precipitation and runoff on streams where the runoff records appear to be above suspicion.

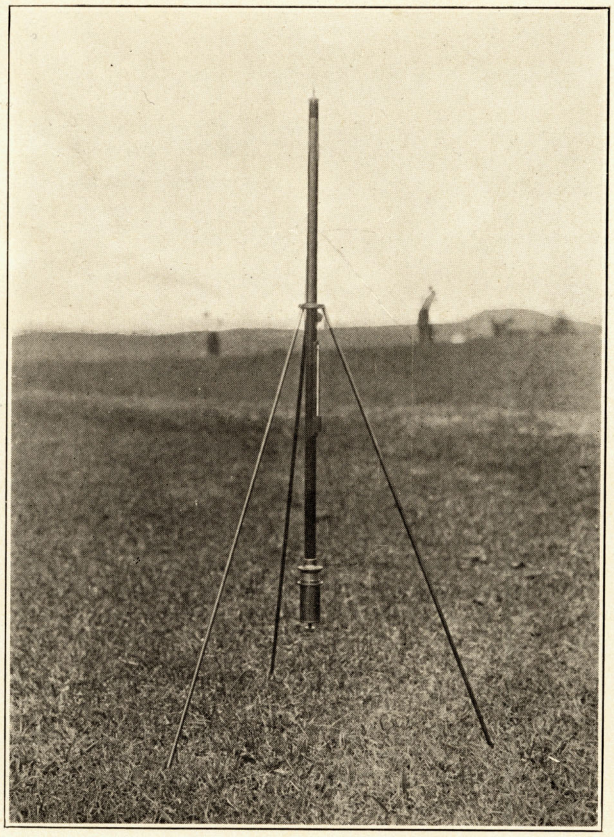

FIG. 1.—"Mountain" barometer and tripod, for use in determining elevations.

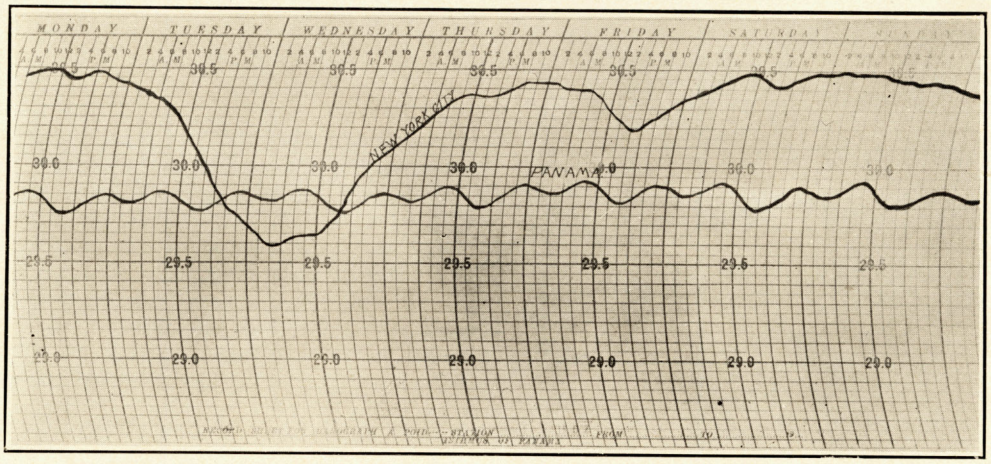

FIG. 2.—Typical winter atmospheric-pressure curves for a 7-day period at New York City and at Panama.

FIG. 1.—Overflow can and snow board. (A, snow board.)

Comparison of snowfall measured by standard rain gage overflow can and by snow board. Horton Hydrologic Laboratory, 1920.

Date, 1920.	Depth on snow board.	Water equivalent of snow.		Ratio: Board can.	Total depth on ground.
		Gage can.	Snow board.		
1	2	3	4	5	6
	Inches.	*Inches.*	*Inches.*		*Inches.*
Jan. 17	5.88	0.142	0.151	1.06	....
19	0.25	.053	.014	0.26	7.50
20	1.00	.041	.041	1.00	8.50
24	4.25	.209	.230	1.10	11.87
25	1.50	.046	.057	1.24	13.00
28	Trace.	.0087	.003	0.34	11.50
Feb. 7	8.00	.544	.600	1.10	20.13
12	0.75	.159	.193	1.21	18.37
16	3.63	.175	.302	1.73	15.00
20	0.19	.018	.018	1.00	....
22	0.75	.022	.029	1.32	....
24	2.38	.071	.091	1.28	....
Total	....	1.489	1.729	1.16	....

ON THE COMPARISON OF METEOROLOGICAL DATA WITH RESULTS OF CHANCE.

By LOUIS BESSON, Chief, Meteorological Service of Paris.

[Translated from the French, and abridged, by Edgar W. Woolard.]

I.

Suppose we have a set of values of some meteorological element, e. g., the mean daily atmospheric pressure for N consecutive days,

$$Y_1,\ Y_2,\ Y_3, \cdot\cdot\cdot\cdot, Y\ .$$

If one of these values is smaller than both the preceding and the following value, then the pressure has passed through a minimum; suppose we find, in the series, M such minima: should we attribute this result to some physical cause which tends to produce M barometric minima in N days, or is it only such that it can be accounted for by the laws of pure chance? We are thus led to the problem of determining how many minima should be found in a series of N numbers obtained by chance selection. (Whatever concerns minima is equally applicable to maxima.)

This problem was investigated some time ago by Grossmann,[1] who employed the method of *variations:* Given a series of numbers, the variation of each is considered to be positive, negative, or zero, according as the following value is larger, smaller, or the same; the numerical values are then discarded, only the signs of the variations being retained. Evidently, a minimum is indicated by a negative variation immediately followed by a positive variation, or separated from the latter by any number of zero variations.

Grossmann derives the formula

$$M=\frac{ab}{N}\left[1+\frac{c}{N-1}+\frac{c(c-1)}{(N-1)(N-2)}+\cdot\cdot\cdot\cdot + \frac{c!}{(N-1)\ (N-2)\ldots\ldots(N-c)}\right],$$

where M is the number of minima which should occur in a series of N chance values, a being the number of positive variations, b that of the negative variations, c that of the zero variations. If there are no zero variations the formula becomes $M=ab/N$; and if, in addition, there are as many positive signs as negative, $M=N/4$.

In reality, the most probable number, k, of mimima in a set of N variations which comprises a positive signs and b negative signs is given by

$$\frac{(a+1)(b+1)}{a+1+b+1}-1<k<\frac{(a+1)\ (b+1)}{a+1+b+1};$$

but this value is practically equal to that given by Grossmann's formula.

II.

If we assume the total number of signs to be infinitely great, it is possible to arrive, by the use of variations, at formulae for the number of minima by a method different from that employed by Grossmann. As a matter of fact, the original problem has no meaning unless we are dealing with a very long series, for the laws of chance are applicable to large numbers only.

Suppose, then, that a sack contains an infinite number of balls marked with positive, negative, and equality signs, in such proportions that for any N balls there will be on the average a positive signs, b negative, and c equality. The total number of balls being infinite, the proportions of the three kinds remaining in the sack after a drawing has been made, no matter how many may already have been drawn, continue to be (a/N), (b/N), and (c/N); then the chances for drawing a positive, negative, or equality sign, are respectively (a/N), (b/N), (c/N); the probability of a minimum with no equality signs therefore becomes (a/N) (b/N), that of a minimum with one equality sign (a/N) (b/N) (c/N), etc. The probability of a minimum is therefore given by

$$\frac{a}{N}\,\frac{b}{N}+\frac{a}{N}\,\frac{b}{N}\,\frac{c}{N}+\frac{a}{N}\,\frac{b}{N}\,\frac{c^2}{N^2}+\cdot\cdot\cdot\cdot=\frac{ab}{N^2}\left(1+\frac{c}{N}+\frac{c^2}{N^2}+\cdot\cdot\cdot\cdot\right),$$

and the number of minima is found by multiplying this probability by N,

$$M=\frac{ab}{N}\left(1+\frac{c}{N}+\frac{c^2}{N^2}+\ldots\ldots\ldots\ldots\right)$$

$$=\frac{ab}{N}\ \frac{1}{1-\dfrac{c}{N}}.$$

This number does not differ sensibly from that given by Grossmann, as long as N is very large. When there are no equality signs, the two formulæ become identical; however, in this case, taking account of the minima which occur complete, for two consecutive signs the probability of a minimum is of course (ab/N^2); there are N–1 pairs of consecutive signs, and the number of minima becomes

$$M=(N-1)\frac{ab}{N^2}.$$

If there are no equality signs in the series, then the shortest possible interval between two consecutive minima is that where the two signs forming a maximum occur between the two signs indicating the minima, thus: $-\ +\ -\ +$. We shall call this a *two-interval;* the probability of the occurrence of such a grouping is $\frac{a^2\,b^2}{N^4}$, and in a set of N signs the probable number of times it should occur would be given by $(N\text{–}3)\frac{a^2\,b^2}{N^4}$, since there are $(N\text{–}3)$ sets of four consecutive signs. Similarly, the next

[1] L. Grossmann, Die Aenderung der Temperature von Tag zu Tag an der deutschen Kuste, aus dem Archiv der Deutschen Seewarte, XXIII Jahrgang, 1900, pp. 34–37.

largest interval, the *three-interval* is caused by either of the groupings $-++-+$, $-+--+$, and hence its probability is

$$\frac{a^3b^2}{N^5}+\frac{a^2b^3}{N^5}=\frac{a^2b^2}{N^4}\left(\frac{a+b}{N}\right)=\frac{a^2b^2}{N^4},$$

or the same as that of the two-interval, and the probable frequency becomes $(N-4)\frac{a^2b^2}{N^4}$.

The *four-interval* is brought about by any one of the three groupings, $-+++-+$, $-+---+$, $-++--+$. The probability is

$$\frac{a^4b^2}{N^6}+\frac{a^2b^4}{N^6}+\frac{a^3b^3}{N^6}=\frac{a^2b^2}{N^4}\quad\frac{a^2+ab+b^2}{N^2},$$

and in a set of N signs it should occur a number of times equal to

$$(N-5)\ \frac{a^2b^2(a^2+ab+b^2)}{N^6}.$$

The law of formation of these expressions is obvious: Generally, the *j-interval* should be encountered in a set of N signs a number of times equal to

$$(N-j-1)\ \frac{a^2b^2(a^{j-2}+a^{j-3}b+\cdots\cdots+b^{j-2})}{N^{j+2}}.$$

If there be equal numbers of positive and negative signs, the probabilities of the respective intervals reduce to

$$\frac{1}{2^4},\ \frac{2}{2^5}\ \left(=\frac{1}{2^4}\right),\ \frac{3}{2^6},\ \frac{4}{2^7},\ \text{etc., and their frequencies to}$$

$$\frac{N-3}{16},\ \frac{N-4}{16},\ \frac{3(N-5)}{64},\ \frac{4(N-6)}{128},\ \text{etc.}$$

III.

There is, however, a second method by which we may arrive at the formulæ required by our original problem.

Let there be a variable, y, susceptible of n different, equally probable, values, $a_1, a_2, \text{------}, a_n$. Assume that y, varying by chance, takes the successive values y', y'', y'''. What is the probability of there being a minimum at y''?

Suppose, first, that n is so great that the relative frequency of the cases where $y''=y'$ or $y''=y'''$ is so small as to be negligible; then there will be a minimum when $y'>y''<y'''$. This double inequality may be satisfied in a number of different ways. If $y''=a_1$, there are $(n-1)$ possible values for y' as well as for y''' (the above series of possible values being arranged in numerical order), giving $(n-1)^2$ possible cases; if $y''=a_2$, there are $(n-2)^2$: if $y''=a_3$, there are $(n-3)^2$; etc. Finally, when y'' has the largest possible value, a_{n-1}, there is but one way of satisfying the inequality. Then the total number of possible ways in which the inequality may be satisfied is

$$(n-1)^2+(n-2)^2+\text{------}+1^2$$

$$=\frac{(n-1)n(2n-1)}{6}$$

$$=\frac{2n^3-3n^2+n}{6}.$$

The probability of the occurrence of a minimum is then the ratio of the number of possible ways in which the inequality may be satisfied to the number of possible cases of different triads; this latter, since to each of the n possible values of y'' correspond n possible values of y' and also of y''', is evidently n^3. The probability therefore becomes

$$\frac{1}{3}-\frac{1}{2n}+\frac{1}{6n^2},$$

or simply $\frac{1}{3}$ if n is infinitely great; then, if y takes on a great number, N, of successive values, there will be $N-2$ *groups* of three successive values and we should find the most probable number of minima to be

$$M=\frac{N-2}{3}.$$

The formula of the method of variations gives as the number of minima

$$M=\frac{N-2}{4}.$$

As is seen, the formulæ given by the two different methods are not the same.

Consider, now, the case where y takes on several successive values which are equal to one another. A minimum remains characterized by three values which we shall continue to designate by y', y'', y''', such that

$$y'>y''<y''',$$

but now y'' is repeated λ times.

The number of possible ways in which the inequality may be satisfied is always

$$\frac{(n-1)n(2n-1)}{6};$$

but now to each of the n^3 possible triads correspond n^λ possible arrangements with the λ extra values of y'' included, giving as the total number of possible different groups $n^{\lambda+3}$. Giving to λ its successive possible values $0, 1, 2, \text{------}$, and summing the *probabilities*, the probability of a minimum becomes

$$\frac{(n-1)n(2n-1)}{6n^3}\left(1+\frac{1}{n}+\frac{1}{n^2}+\cdots\cdots\right)$$

$$=\frac{(n-1)n(2n-1)}{6n^3}\ \frac{n}{n-1}=\frac{n^2(2n-1)}{6n^3}$$

$$=\frac{1}{3}-\frac{1}{6n}.$$

It is sufficient to multiply this probability by N in order to get the most probable *number* of minima one should encounter in a set of N values of y; this follows from the mathematical definition of probability. Then when n is infinitely great the *probability* is $\frac{1}{3}$, the same as found before.

FREQUENCY OF VARIOUS INTERVALS BETWEEN TWO CONSECUTIVE MINIMA—SECOND METHOD.

Assume, first, that n is so great that we may neglect the minima in which several equal values occur.

Minimum followed by a given value.—Consider the group of three consecutive values, y', y'', y''', such that $y'>y''<y'''$, and let y''' be given equal to a_p; then y'' can take any one of the values $a_1, a_2, \text{-------}\ a_{p-1}$; if $y''=a_1$, y' can take any one of the remaining

$n-1$ higher values; if $y''=a_3$, y' can take any one of the other $n-2$ values; and so on, until when $y''=a_{p-1}$, y' has $n-p+1$ possible values. The *total* number of possible arrangements is

$$(n-1)+(n-2)+\ldots\ldots\ldots+(n-p+1).$$

This sum is a function of p; it becomes equal to $(n-1)$ when $p=2$, to $(n-1)+(n-2)$ when $p=3$, etc. Let

$$(n-1)+(n-2)+\ldots\ldots\ldots+(n-p+1)=f_1(p).$$

Minimum followed by two rises, of which the second can be zero, the series ending in a given value.—Consider the group of four consecutive values, y', y'', y''', y^{iv}, such that $y'>y''<y'''<y^{iv}$, and let y^{iv} be given equal to a_p.

Now, y''' can take any of the values $a_1, a_2, \cdots a_p$. It is easily seen that the total number of possible arrangements giving us our required type of minimum is

$$f_1(2)+f_1(3)+\cdots+f_1(p).$$

Now let

$$f_1(2)+f_1(3)+\cdots+f_1(p)=f_2(p).$$

In the same way, applying the last formula to the case of a minimum followed by three rises, of which the last two may be zero, the entire series ending in the value a_p, we find as the total number of possible ways in which such can be effected

$$f_2(2)+f_2(3)+\cdots f_2(p)=f_3(p);$$

and, generally, for a minimum followed by q rises, the last q-1 of which may be zero, the series ending in the value a_p, this number is

$$f_q(p).$$

The same reasoning evidently applies to a minimum preceded by q rises, the first q-1 of which may be zero, the initial value being a_p.

The value of $f_1(p)$ can readily be calculated with the help of the well-known formulas for the sums of the first n integers, their squares, cubes, etc.

We have

$$f_1(p)=\frac{-p^2+(2n+1)p-2n}{2};$$

$$f_2(p)=\Sigma_2^p f_1(p)$$

$$=\frac{-\Sigma_2^p p^2+(2n+1)\Sigma_2^p p-2n(n-1)}{2}$$

$$=\frac{-p^3+3np^2-(3n-1)p}{6};$$

$$f_3(p)=\frac{-p^4+(4n-2)p^3+p^2-(4n-2)p}{24},$$

etc.

The probabilities of the various intervals can now be calculated.

The two-interval.—In the notation of the method of investigation now being employed, the *two-interval* is given by a group of five consecutive values, y', y'', y''', y^{iv}, y^{v}, which satisfy the conditions

$$y'>y''<y'''>y^{iv}<y^{v}.$$

A minimum immediately follows and precedes y''' simultaneously.

If $y'''=a_p$ the total number of possible ways of satisfying the above conditions is $[f_1(p)]^2$; but y''' can take any of the values $a_2, a_3, \cdots a_n$; hence the total number of possibilities becomes

$$\Sigma_{p=2}^{p=n}[f_1(p)]^2=\Sigma_{p=2}^{p=n}\frac{[(2n+1)p-p^2-2n]^2}{4}$$

$$=\frac{1}{30}(4n^5-10n^4+10n^3-5n^2+n).$$

The total number of possible sets of five values being n^5, the probability of a minimum of this type is

$$\frac{1}{30}\left(4-\frac{10}{n}+\frac{10}{n^2}-\frac{5}{n^3}+\frac{1}{n^4}\right),$$

or, since n is assumed indefinitely large, simply 2/15. Since in a set of N values of y there are N-4 groups of five values, the most probable number of two-intervals is

$$\frac{2}{15}(N-4).$$

The three-interval.—This interval is characterized by a group of six consecutive values which satisfy the conditions

$$y'>y''<y''',$$

and

$$y^{iv}>y^{v}<y^{vi}.$$

Between the two minima we may have a rise and two falls, or two rises and a fall; the second of the two rises and the first of the two falls may be zero.

We now get as the number of ways in which these conditions can be met

$$\Sigma_{p=2}^{p=n}f_1(p)f_2(p)+\Sigma_{p=2}^{p=n}f_2(p)f_1(p)$$

$$-\Sigma_{p=2}^{p=n}[f_1(p)]^3.$$

The negative term represents the number of cases in which $y'''=y^{iv}$, these being counted twice in the preceding terms; it is negligible in comparison with the others because it gives a polynomial of degree 5 in n, and when the above expression is divided by n^6 to get the probability of the three-interval the quotient becomes zero when n becomes indefinitely great. The probability sought for is therefore the coefficient of the term in n^6 in the expression

$$2\Sigma_{p=2}^{p=n}f_1(p)f_2(p).$$

The value can, however, be found at once without calculation by noting that because of the mutual independence of y''' and y^{iv} the probability of the three interval is simply the square of that of a minimum, or 1/9; then in a set of N values, one should find $\frac{1}{9}(N-5)$ three intervals.

Now consider $j+3$ successive values of y, of which each may be indifferently $a_1, a_2, \ldots\ldots, a_n$, thus making in all n^{j+3} possible sets. How many of these possible sets will have minima in y'' and in y^{j+2}, and only in those places?

Between the two minima separated by the j-interval there can be one rise and j-1 falls, two rises and j-2 falls, and so on up to j-1 rises and one fall; any of these may be zero with the exception of the first rise and the last fall.

The various cases furnish in all a number of possibilities given by

$$\Sigma_{p=s}^{p=n}(f_1 f_{j-1}+f_2 f_{j-2}+\cdots\cdots+f_{j-1}f_1),$$

in which the terms equidistant from the ends are equal. The probability of the j-interval is obtained by dividing this expression by n^{j+3}.

Strictly speaking, some cases are counted several times in the above formula, viz, those in which the highest value between the two minima is repeated several times; it is easily seen, however, that if n be very large, the number of such cases is negligible relatively to that of the others; and that for n infinite, the *probability* of the interval is given simply by the coefficient of n^{j+3} in the polynomial in n by which the above sum is expressed.

It is perfectly practicable to calculate the value of this development; the computations for the intervals four, five, six, seven, and eight give for the respective *probabilities*

$$\frac{2}{35},\ \frac{1}{45},\ \frac{4}{567},\ \frac{1}{525},\ \frac{2}{4455}.$$

Therefore, in a series of N values of y, there should be found

$\frac{2}{35}(N-6)$ *four-intervals,*
$\frac{1}{45}(N-7)$ *five-intervals,*
$\frac{4}{567}(N-8)$ *six-intervals,*
$\frac{1}{525}(N-9)$ *seven-intervals,*
$\frac{2}{4455}(N-10)$ *eight-intervals.*

These results are in complete disagreement with those reached by the method of variations. We shall investigate the probable cause of the discrepancy later on.

IV.

All of the preceding work refers to the case where the various possible values of y are equally likely to occur.

Suppose, however, that the variable y satisfies the well known law of Gauss or Normal Error Law; i. e., if a be the mean value, and

$$z=y-a,$$

then the probability of a value of y being such that z is comprised within the limits z and $z+dz$ is

$$\frac{h}{\sqrt{\pi}}e^{-h^2z^2}dz,$$

h being a constant peculiar to the variable.

The probability of a value z'' being preceded by a greater value z' is evidently given by

$$\frac{h}{\sqrt{\pi}}\int_{z''}^{+\infty}e^{-h^2z'^2}dz'.$$

Put

$$\frac{h}{\sqrt{\pi}}\int e^{-h^2z^2}dz=u,$$

$$\frac{h}{\sqrt{\pi}}\int e^{-h^2z'^2}dz'=u',$$

$$\frac{h}{\sqrt{\pi}}\int e^{-h^2z''^2}dz''=u'',$$

etc.

It is known that the indefinite integral u has the value $\frac{1}{2}$ for $z=+\infty$, and $-\frac{1}{2}$ for $z=-\infty$. The probability sought for may be written

$$u'_{+\infty}-u'',$$

or

$$\tfrac{1}{2}(-2u''+1).$$

Now let us denote by P_1 the probability that a value z''' will be preceded by a fall which is itself preceded by a rise, or in other words that z''' will be preceded by a minimum in z''. We have just calculated the probability of a value z' preceding a lower value z''; the probability of a value falling between the limits z'' and $z''+dz''$ is

$$\frac{h}{\sqrt{\pi}}e^{-h^2z''^2}dz'',\text{ or }du''.$$

In order to fulfill the conditions of our problem, z'' can take on any value between z''' and $-\infty$; hence, the probability sought for will be given upon multiplying together the two above probabilities and summing from $-\infty$ to z''':

$$P_1=\int_{-\infty}^{z'''}(\tfrac{1}{2}-u'')\,du''$$

$$=\tfrac{1}{8}\,(-4u'''^2+4u'''+3).$$

Denoting by P_2 the probability that a value z^{iv} will be preceded by a minimum in z'' followed by two rises, we find in the same manner

$$P_2=\tfrac{1}{48}(-8u^{iv3}+12u^{iv2}+18u^{iv}+5).$$

We see that this last expression can be deduced from the preceding one by multiplying the numerical factor by 1/6, and the coefficients of the terms in the polynomial by 6/3, 6/2, and 6/1, then annexing the final term, 5.

If we work out the problem of the probability P_3 of a minimum followed by three rises, the numerical factor of P_2 is multiplied by 1/8, the coefficients by 8/4, 8/3, 8/2, 8/1, and the final term is 7.

We can easily determine the law of formation of these expressions:

In the expression giving P_{i+1}, the numerical factor is equal to the product of that of P_i by $\frac{1}{2i+2}$; the coefficients of the terms in u are equal to those in P_i multiplied by the fractions which have for numerators $2i+2$, and for denominators the successive integers decreasing to 1; the final term is $2i+1$.

It may be remarked that the probability of a value z being followed by i falls, then by a minimum, is the same as the probability of a value z being preceded by a minimum and i rises.

The probabilities of a minimum and of various intervals between minima may now be found.

The probability of a value falling between z and $z+dz$ is du; the probability that this value will be preceded by a fall, itself preceded by a rise, is

$$P_1=\tfrac{1}{8}\,(-4u^2+4u+3)$$

Multiplying the two probabilities together, then summing from $-\infty$ to $+\infty$, since z has this range of values, the probability of a minimum becomes

$$\tfrac{1}{8}\int_{-\infty}^{+\infty}(-4u^2+4u+3)\,du$$

$$=\tfrac{1}{3}.$$

Thus, the probability of a minimum does not depend upon h, and it is the same as in the case where all the values of the variable have equal probabilities.

Reasoning as before, we find that the probability of two consecutive minima being separated by the j-interval is given by the expression

$$\int_{-\infty}^{+\infty}(P_1\,P_{j-1}+P_2\,P_{j-2}+\cdots\cdots+P_{j-1}\,P_1)du.$$

If the calculations be carried out, and j given successively the values 2, 3, ... etc., the same numbers are found as before. The same formulæ for the number and the distribution of minima, therefore, apply to both a variable which obeys the law of Gauss and one of which all values are equally probable.

V.

Three different tests of both sets of formulæ were carried out by counting the minima in sets of numbers which clearly were obtained by pure chance. The results left no doubt but that Grossmann's formulæ were in error, and that those of the second method were very close to the truth. Grossmann's formulæ should not, therefore, be employed in meteorological work, for it might lead us to attribute to some systematic influence the departures which would in reality be due only to the workings of chance.

The discrepancy between the two sets of formulæ may, perhaps, be explained as follows: We are considering a variable which may oscillate between two extreme limits. After taking on a small value, it has greater chances of increasing than of further diminishing, i. e., the probability of a positive sign is greater than that of a negative sign; the opposite is true after the occurrence of a large value. In Grossmann's theory, however, the probabilities of the two signs remain constant. Therefore, however paradoxical it may seem, if a variable is increasing and diminishing purely at random, it does not follow that the succession of signs of its variations will obey the laws of chance.

Grossmann considered also the probability of more or less prolonged rises or falls, and his methods and results have been used by other meteorologists; our second method leads, however, to entirely different formulæ.

Suppose the variable follows the law of Gauss; the formulæ will also be applicable when the variable has a large number of equally probable values.

Probability of an isolated rise.—This condition is given by four consecutive values such that

$$z' > z''' < z''' > z^{\text{iv}}.$$

The probability that z''' will be preceded by a fall, then by a rise, has been calculated to be

$$P_1=\frac{1}{8}(-4u'''^2+4u'''+3).$$

The probability that, in addition, z''' be superior to z^{iv} may be obtained by multiplying P_1 by the elementary probability of z''', viz, du''', and summing from z^{iv} to ∞:

$$\varphi_1=\frac{1}{8}\int_{z^{\text{iv}}}^{+\infty}(-4u'''^2+4u'''+3)du$$

$$=\frac{1}{48}(8u^{\text{iv}3}-12u^{\text{iv}2}-18u^{\text{iv}}+11).$$

Since z^{iv} can now be anything from $-\infty$ to $+\infty$, we must sum $\varphi_1\,du^{\text{iv}}$ between those limits, giving as our required probability

$$\int_{-\infty}^{+\infty}\varphi_1 du^{\text{iv}}=\frac{5}{24}.$$

Probability of a series of two rises.—Such a series is given by the grouping

$$z' > z'' < z''' < z^{\text{iv}} > z^{\text{v}}.$$

The probability that z^{iv} will be preceded by a fall, then by two rises, is,

$$P_3=\frac{1}{48}(-8u^{\text{iv}3}+12u^{\text{iv}2}+18u^{\text{iv}}+5).$$

The probability that, in addition, z^{iv} will be greater than z^{v} is

$$\varphi_2=\frac{1}{48}\int_{z^{\text{v}}}^{+\infty}(-8u^{\text{iv}3}+12u^{\text{iv}2}+18u^{\text{iv}}+5)du$$

$$=\frac{1}{384}\,(16u^{\text{v}4}-32u^{\text{v}3}-72u^{\text{v}2}-40u^{\text{v}}+41).$$

Finally, as z^{v} can take on any value between $-\infty$ and $+\infty$, the required probability is

$$\int_{-\infty}^{+\infty}\varphi^2\,du^{\text{v}}=\frac{11}{120}.$$

These calculations may be extended to the higher cases with the aid of the already formed expressions for P_3, P_4, etc.

It is thus found that in a set of N values there should be found, as the most probable number,

$\frac{5}{24}(N-3)$ single rises or falls,
$\frac{11}{120}(N-4)$ series of 2 rises or falls,
$\frac{19}{720}(N-5)$ series of 3 rises or falls,
$\frac{29}{5040}(N-6)$ series of 4 rises or falls,
$\frac{41}{40320}(N-7)$ series of 5 rises or falls,
$\frac{55}{362880}(N-8)$ series of 6 rises or falls,
$\frac{71}{3628800}(N-9)$ series of 7 rises or falls.

The fractions figuring in the above table may be deduced one from another in a simple manner:

The numerators increase successively by 6, 8, 10, 12, etc., and the denominators are multiplied successively by 5, 6, 7, 8, etc. The list of probabilities can therefore easily be recalled and extended. A verification of the preceding formulæ was made.

VI.

The formulæ derived in this paper appear to satisfy all demands that can be made upon them for their application to meteorology.

For a rigorous comparison it would be necessary, in any particular case, first of all to give the mathematical variable all the characteristics possessed by the natural variable being considered, and leave undetermined only its tendency to increase or decrease in value, this last being left to the domain of chance. It is then a question whether or not our formulæ apply to a variable of the nature so determined, so that we may legitimately compare the mathematical results of chance with the observational results of experience, and decide as to the existence of a systematic influence.

It seems to the writer that the only things which it is necessary to consider in this connection are (a), the law of probability for the occurrence of the various possible values, and (*b*), the mean variability, i. e., the mean difference between successive absolute values of the variable.

(*a*) Our formulæ are exact in both the very different cases where all values are equally probable and where they follow the law of Gauss, as well as in still different cases. Hence, it can not be doubted that they possess a very great degree of generality, and no matter what law which the quantities occurring in meteorological applications might follow, the application of these formulæ would not lead us into serious error; as a matter of fact, such quantities usually follow the law of Gauss quite closely.

(*b*) The successive values of the mathematical variable are independent of one another, and the mean variability is $\sqrt{2}$ times the mean departure; but for the meteorological elements, particularly for a series of successive daily values, the values are not independent, and the mean variability is usually somewhat less than the above quantity.[1] Hence we should impose on the free mathematical variable the supplomentary condition that it have the same mean variability as has the element being considered: unfortunately, one encounters here a mathematical difficulty (also met with in the theory of an imperfect gas) which has not yet been surmounted.

If we arbitrarily fixed the mean variability it would amount to admitting that the probability of the occurrence of a value is a function of the preceding value, which is exactly contrary to the fundamental assumptions upon which the theory of probability rests, and according to which all our formulæ have been derived.

However, the mean variability does not play such an important rôle as it would at first sight seem to; and some simple considerations show that the introduction of a condition reducing the mean variability somewhat would not modify the indications of our formulæ.

[1] Ch. Goutereau, Sur la variabilité de la température, Annuaire de la Société météorologique de France, 1906, p. 122.

WATERSPOUTS ON THE SOUTHERN CALIFORNIA COAST.

"Visitors at the beaches (Port San Luis, Avila, Pismo, and Oceano) Sunday afternoon about 5 o'clock had the opportunity of observing," says the San Luis Obispo Tribune, "a most unusual phenomenon, that of an immense waterspout traveling at a high rate of speed toward the beach. The spout was shaped like a funnel, and is said to have been about 2,000 feet high, extending as high as the clouds and spreading out into a fine mist. The spout was first visible from Avila and Pismo when it was about 4 or 5 miles distant from the shore, and from that time traveled rapidly until it broke on the beach between Shell Beach and the old Oilport refinery.

"Fishermen who landed at Avila later in the evening stated that they had seen three spouts at one time, two of which were traveling in the direction of the Pecho and one toward Shell Beach. The largest of the spouts was one of those going toward Pecho, and probably covered about 5 acres in area, according to the fishermen."

In describing these waterspouts the San Francisco Call says:

> The phenomenon was followed by a tremendous downpour of rain. Fishermen at sea north of the port viewed the three spouts simultaneously. As they approached the shore the two larger ones mounted the headlands, but the third was diverted. It swept around the buoy and proceeded across the bay at a speed estimated to be in excess of 40 miles an hour.
>
> Water within a diameter of from 150 to 200 yards was violently agitated and appeared to be siphoned upward to a mass of clouds some 2,000 feet above. As the spout approached the shore persons near state that there was a tremendous roar. The funnel apparently detached itself from the water and was drawn gradually up into the mass of overhanging, rapidly moving, cumulo-nimbus clouds. Violent gusts of wind followed the appearance of the spouts and continued for several hours.
>
> According to J. E. Hissong, United States weather observer here, the spouts were due to the overrunning of surface air by a layer of colder air as weather control passed from an area of low to an area of high pressure.
>
> The spouts are the first to appear on this coast within the memory of the oldest inhabitant.

INFLUENCE OF THE WIND ON THE MOVEMENTS OF INSECTS.

By WILLIS EDWIN HURD.

[Weather Bureau, Washington, Jan. 6, 1920.]

The weather perhaps has more to do with the control of insect life than all other factors combined, and the significance of this meteorological aspect is varied. Cold, heat, rain, hail, humidity, drought, sunshine, electricity, and wind are factors. Temperature, rain, and wind movement are of utmost economic importance. Sudden cold and rain in early summer may more or less completely destroy the incubating or newly hatched members of what would otherwise prove to be a vast swarm of destructive crop eaters. Drought may retard or destroy numbers of insects in their metamorphoses. Frosts at the moment of appearance of the imago may wreak untold disaster to the tender brood. And prevailing winds may so accelerate or retard the direction of movement of many injurious species at the time of their seasonal advance as to cause or avert great economic disasters.

Thus the winds may upon occasions become the paramount issue of life or death for the little fliers of our fields and orchards. When we see butterflies and other large-winged, small-bodied insects fluttering hither and yon, buffeted about in the air on a windy day, the impression is strong that any extended flight of such creatures must conform with the direction of the wind. And yet the facts do not always bear out such a conclusion, since in reality the unsteady butterfly is much more capable of forcing itself against the air current than is the heavy locust or the more projectile-like beetle.

The dispersion of insects by means of winds is a matter of constantly increasing agricultural importance. It interests the farmer inasmuch as it may affect his crops; and as it affects the agricultural staples, so does it vitally interest all of us, who need to be fed and clothed. The question was formerly more discussed by the student of geographical zoology, as it affected his plan regarding the spread of a type of life from region to region within coasts or across the seas. The South American locust, for instance, is believed in some scientific circles to be descended from the survivors of an African swarm of identical genus which, following more or less passively in the steady currents of the northeast trades, succeeded in crossing the Atlantic Ocean.

The flights, or migrations, may be largely voluntary, though a good percentage are quite the opposite. In nearly all cases the winds play an important part, and most insects are likely to follow the direction of the air currents, although some are inclined to quarter the